Domain wall formation in the Cahn-Hilliard-Cook equation

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Formation of domain walls during phase-separating transition in the Cahn-Hilliard-Cook equation is studied. Density of domain wall scales like a sixth root of quench rate for equal concentrations and like a square root of quench rate for unequal concentrations of components. For a slow inhomogeneous transition, the density is linear in a velocity of temperature front.

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I. INTRODUCTION

We address the question of domain wall formation during a phase-separating transition in binary systems with conserved order parameter. Studies of defect formation during a rapid phase transition were initiated in the context of cosmology. It has been pointed out that topological defects that formed during subsequent symmetry breaking phase transitions could provide seeds for structure formation in the early universe. Kibble [1] gave a detailed theory of defect formation in first-order phase transitions that proceed by bubble nucleation and general geodesic argument that relates density of topological defects to the density of domains with disoriented order parameter. The theory was supplemented by Zurek [2] with a scenario for second-order transitions. The second-order transitions proceed by spinodal decomposition. The slower the rate of the transition, the longer is the characteristic length scale frozen into the system after the quench. Kibble-Zurek (KZ) scenario was tested in a number of condensed-matter experiments [3].

In cosmological transitions, the order parameter is not expected to be conserved. That is why defect formation in transitions with conserved order parameter was not studied in the cosmological context. However, order parameter is conserved in a wide class of systems such as binary alloys, copolymers, and binary fluids and the phase-separating transition in these systems is of second order. It turns out that for equal concentrations of the binary components the density of defects can be predicted by an argument that is a direct generalization of the KZ argument. The scaling between the transition rate and the characteristic length turns out to be different than that for a nonconserved order parameter. The qualitative similarity between conserved and nonconserved cases for equal concentrations suggest the binary systems as another experimental testing ground for the KZ paradigm.

The order parameter conservation introduces phenomena that are interesting in their own right. They show up when the two components of the binary mixture differ in concentration. This difference of concentrations translates into a nonzero average order parameter. In this case, defect formation can be divided into two stages. The first stage is again well described by the KZ scenario. KZ mechanism predicts a characteristic length scale frozen into the fluctuations around the biased nonzero order parameter. In the second stage a selection of domains takes place such that in the end one obtains bubbles of the minority phase scattered in the majority phase. The size of the bubbles is determined by the KZ length scale but the density of the bubbles scales with a different exponent than the KZ length scale. This discrepancy is necessary to conserve the order parameter.

As we will discuss in some detail in Sec. V, hydrodynamic effects are irrelevant for the formation of initial domains during the transition. Hydrodynamics can step in later on during subsequent phase ordering kinetic process. In this paper we predict the initial density of domain wall just after the transition but before the phase ordering kinetics steps in. When we neglect hydrodynamic flows, then the dynamics of phase separation can be described by the celebrated Cahn-Hilliard-Cook equation (CHCE) for conserved real order parameter ϕ

$$\dot{\phi}=\nabla^2[-\epsilon(t,\vec{x})\phi+\phi^3-\nabla^2\phi]+\vec{\nabla}\vec{\xi}, \tag{1}$$

where an overdot$=\partial_t$. We allow for variation of the symmetry breaking parameter ϵ (temperature/pressure) both in space and in time. $\vec{\xi}$ is assumed to be a vector of white Gaussian noises with correlators

$$\langle\xi^a(t,\vec{x})\rangle=0,$$
$$\langle\xi^a(t_1,\vec{x}_1)\xi^b(t_2,\vec{x}_2)\rangle=2T\delta^{ab}\delta(t_1-t_2)\delta(\vec{x}_1-\vec{x}_2), \tag{2}$$

where the indices a,b run from 1 to the number of spatial dimensions d.

II. TRANSITION WITH EQUAL CONCENTRATIONS

To begin with let us consider a uniform linear phase transition with

$$\epsilon(t,\vec{x})=\frac{t}{\tau}, \tag{3}$$

with $\phi=0$ on average at the initial $t=-\infty$, which is preserved by CHCE evolution. Any uniform transition close to the critical point at $\epsilon=0$ can be described by such a linearized $\epsilon(t)$.

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For $t<0$, the system is in a symmetric phase; ϕ is subject to small fluctuations around 0. For this stage of the quench and also for the onset of spinodal instability, just after ϵ crossed 0, Eq. (1) can be linearized in ϕ,

$$\dot{\phi}=\left[-\frac{t}{\tau}+3\langle\phi^2\rangle\right]\nabla^2\phi-\nabla^4\phi+\nabla\vec{\xi}. \tag{4}$$

The mean-field $\langle\phi^2\rangle$ is kept just to control validity of the linearization. We solve Eq. (4) by neglecting the mean-field term and performing Fourier transformation

$$\phi(t,\vec{x})=\int_{-\infty}^{+\infty}d\vec{k}e^{i\vec{k}\vec{x}}\phi(t,\vec{k}). \tag{5}$$

The Fourier transformed linearized equation (4) is

$$\dot{\phi}(t,\vec{k})=\frac{t}{\tau}k^2\phi(t,\vec{k})-k^4\phi+i\vec{k}\vec{\xi}(t,\vec{k}). \tag{6}$$

Even before we proceed with a formal solution, we can give an intuitive Kibble-Zurek-like argument to estimate the characteristic length scale after the quench. All the Fourier modes fluctuating according to Eq. (6) are stable as long as $t<0$. For $t>0$, long-wavelength modes become unstable and they begin to grow exponentially. The time scale τ_k for this exponential growth of the mode $\vec{k}$ follows from Eq. (6),

$$\frac{1}{\tau_k}=\frac{t}{\tau}k^2-k^4. \tag{7}$$

We are interested in small k, where τ_k is positive. τ_k is minimal and the instability is the strongest for $k_t^2=t/2\tau$. The minimal τ_k is $\tau_t=4\tau^2/(t^2)$. The fluctuations begin to blow up exponentially at time $\hat{t}\sim\tau^{2/3}$ when $\tau_{\hat{t}}\sim\hat{t}$. At $\hat{t}$ the instability is the strongest for the modes with $\hat{k}\sim 1/\tau^{1/6}$ that define a characteristic length scale

$$\hat{\xi}\sim\tau^{1/6}. \tag{8}$$

This length scale dominates the exponentially growing fluctuations when they enter the nonlinear regime around the time $\hat{t}$. At $\hat{t}$ the width of the domain wall (the healing length) is also proportional to $\tau^{1/6}$, compare Eq. (1), so the $\hat{\xi}\sim\tau^{1/6}$ is the only relevant length scale. In the nonlinear regime, the fluctuations saturate in the form of positive-ϕ and negative-ϕ domains separated by domain walls. $\hat{\xi}$ determines the average size of the domains and the density of domain walls.

We will confirm this prediction by a more detailed calculation. Equation (6) can be solved for any $\vec{k}$ with the help of its Green function and the correlations (2). The ''power spectrum'' $P(t,\vec{k})$ of the fluctuations is

$$\langle\phi^\star(t,\vec{k})\phi(t,\vec{p})\rangle\equiv P(t,\vec{k})\,\delta(\vec{k}-\vec{p}),$$

$$P(t,\vec{k})=\frac{T\sqrt{\tau}k}{\pi}e^{k^2(t-\tau k^2)^2/\tau}\int_{-\infty}^{k(t-\tau k^2)/\sqrt{\tau}}ds e^{-s^2}. \tag{9}$$

The fluctuations, measured by $\langle\phi^2\rangle=\int d\vec{k}P(t,\vec{k})$, are small for $t<0$. For $t>0$, they start to blow up exponentially and at $\hat{t}>0$, the linear approximation involved in solving Eq. (4) breaks down,

$$3\langle\phi^2\rangle\equiv 3\int d\vec{k}P(\hat{t},\vec{k})\approx\frac{\hat{t}}{\tau}. \tag{10}$$

For $t>0$ and sufficiently large the error function integral in Eq. (9) is constant for small k^2 (but it is still essential to suppress the divergence for large k^2). The remaining exponent is peaked at $k^2\approx t/3\tau$ with a maximum of $\exp(4t^3/27\tau^2)$. This maximum begins to blow up at $\hat{t}\sim\tau^{2/3}$. It is around this time that $3\langle\phi^2\rangle$ passes through t/τ and the linear approximation in Eq. (4) breaks down: domain walls of width $\sim\tau^{1/6}$ begin to form. Fluctuations with wavelengths shorter than the domain wall width, or $k>\hat{k}=\tau^{-1/6}$, are irrelevant for domain wall formation. Density of domain wall that are going to form can be identified with the density of zeros of $\phi(\hat{t},\vec{x})$ smoothed over $k>\hat{k}$, which is, according to a formula from [4],

$$n=\frac{\pi}{2}\sqrt{\frac{\int_{k<\hat{k}}d\vec{k}k^2P(\hat{t},\vec{k})}{\int_{k<\hat{k}}d\vec{k}P(\hat{t},\vec{k})}}. \tag{11}$$

If we draw a straight line through the system, this formula tells us how often the line crosses a domain wall. After introducing an integration variable $\vec{k}/\hat{k}$ one can see that any τ dependence can be factorized in front of the integrals so that

$$n\sim\frac{1}{\tau^{1/6}} \tag{12}$$

for any τ.

We performed numerical simulations in one spatial dimension of the fully nonlinear Eqs. (1), (2), and (3). The density of kinks and antikinks is consistent with the prediction (12) (compare Fig. 1).

In the limit of adiabatic transition, $\tau\to\infty$, the system stays close to the critical point for a time long enough to coarsen at long distance. For a fast quench, substantial amount of initial disorder is frozen into the ordered phase in the form of domain walls. In one dimension, the domain walls (kinks) are permanent records of the transition in a sense that kink-antikink pairs coarsen only logarithmically with time [6]. In more than one dimension, Eq. (12) gives an initial density of domain wall that is later gradually eradicated by phase ordering kinetics.

III. TRANSITION WITH UNEQUAL CONCENTRATIONS

A. Transition through spinodal decomposition

The two phase-separating phases may differ by average concentration. In that case the average conserved ϕ is $M\neq 0$. We take $M>0$ for definitness. The mixed phase is a uniform $\phi=M$ plus small fluctuations driven by noise. This

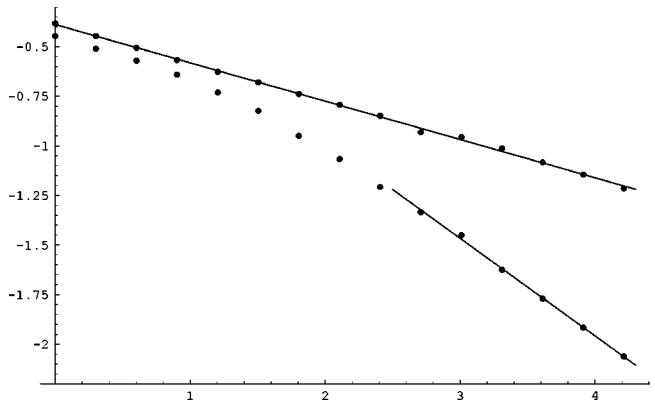

FIG. 1. $\log_{10}(n)$ as a function of $\log_{10}(\tau)$ for $M=0$ (the top plot) and for $M=2$ (the bottom plot) according to numerical simulations. For $M=0$, the slope is 0.18 ± 0.01 as compared to the theoretical $1/6\approx0.17$. For $M=2$, the slope saturates for $\log_{10}(\tau)>2$ at 0.48 ± 0.05 as compared to the theoretical 0.50. At low τ, the $M=0,2$ results tend to be the same. Vertical size of a point is its statistical error. Simulations were done at $T=10^{-5}$ on a $\Delta x=1$, $\Delta t=0.01$ lattice of size 1024 with periodic boundary conditions. $\epsilon(t)$ was swept from $3M^2-10\tau^{-1/3}$ to $3M^2+10\tau^{-1/3}$. Kinks were counted at final time. Density n is an average over many runs.

phase is stable for $\epsilon<0$. As the system is driven through the transition, ϵ increases and eventually becomes positive. Unlike for $M=0$ (see Sec. II), the uniform $\phi=M$ remains stable in a range of $\epsilon>0$.

According to Eq. (1), for $\epsilon>0$, the field ϕ lives in a double-well potential

$$V(\phi)=-\frac{\epsilon}{2}\phi^2+\frac{1}{4}\phi^4. \tag{13}$$

As we can find by linearizing this potential for small fluctuations around the uniform $\phi=M$, $\phi=M$ configuration is stable when $\epsilon<3M^2$. For $\phi=M$ to be stable, M has to lie outside the two inflection points $\phi=\pm\sqrt{\epsilon/3}$ of the double-well potential $V(\phi)$, where $V(\phi)$ is a convex function. In the process of the quench, $\epsilon(t)$ increases. The system enters the transition in a state of uniform $\phi=M$ with small fluctuations on top of it that are driven by noise. As long as $\epsilon<3M^2$, the uniform $\phi=M$ is stable against small perturbations.

When ϵ crosses $3M^2$, the long wavelength modes become unstable in a very similar way as for $M=0$. For any smooth quench, $\epsilon(t)$ can be linearized around $3M^2$ as

$$\epsilon(t)=3M^2+t/\tau. \tag{14}$$

At the same time we can expand

$$\phi(t,\vec{x})=M+\tilde{\phi}(t,\vec{x}) \tag{15}$$

for any small fluctuation field $\tilde{\phi}(t,\vec{x})$. Equation (1) when linearized in $\tilde{\phi}$ gives

$$\dot{\tilde{\phi}}=-\frac{t}{\tau}\nabla^2\tilde{\phi}-\nabla^4\tilde{\phi}+\nabla\vec{\xi}, \tag{16}$$

which is formally the same as Eq. (4). Because of this formal similarity, the length scale $\hat{\xi}\sim\tau^{1/6}$ frozen into the exponentially blowing up fluctuations is the same as for $M=0$. However, unlike for $M=0$ this is not the end of the story.

At $\hat{t}$, the fluctuations $\tilde{\phi}$ around $\phi=M$ have a characteristic length scale $\hat{\xi}$. $\tilde{\phi}(\hat{t},\vec{x})$ can be seen as a mosaic of positive $\tilde{\phi}$ and negative $\tilde{\phi}$ ''domains'' with a characteristic volume of $\hat{\xi}^d$. $\tilde{\phi}$ in these domains is very small; it is far from saturation, so the domains are just potential seeds for fully fledged domains with large saturated order parameter. If there were no ϕ conservation, then $\tilde{\phi}$ in these $\pm$ seeds would quickly grow until it crossed the positive/negative inflection point of the potential $V(\phi)$. After that, the growth would slow down because $V(\phi)$ is convex beyond its inflection points. The domains would gradually saturate with their ϕ's close to the minima of $V(\phi)$. However, such an unbiased pattern of positive and negative domains would have $\phi=0$ on average, which is not consistent with the initial $\phi=M$.

The solution to this problem is that $\tilde{\phi}$ in some (not sufficiently) negative seeds will not manage to hop to the negative inflection point. They will go to the positive inflection point. A fraction q of negative seeds that will grow negative can be estimated as follows: A fraction q of negative seeds ends at the negative inflection point where $\phi\approx-\sqrt{M^2+1/3\tau^{1/3}}$ at $t\approx\hat{t}$. The fraction 1 of the positive seeds and a fraction $(1-q)$ of the negative seeds will end at the positive inflection point where $\phi\approx+\sqrt{M^2+1/3\tau^{1/3}}$. To keep the final $\phi=M$ on average, q must satisfy

$$M=\frac{-q+[1+(1-q)]}{2}\sqrt{M^2+\frac{1}{3\tau^{1/3}}}. \tag{17}$$

For $M^2\tau^{1/3}\gg1$ we obtain $q\sim1/M^2\tau^{1/3}$. Given that the average linear size of the seeds is $\hat{\xi}$ the average linear density of domain walls (defined by the average density of intersections between a strait line and domain walls) should scale like $q/\hat{\xi}$,

$$n\sim q/\hat{\xi}\sim\frac{1}{M^2\tau^{1/2}}\quad\text{for}\quad M^2\tau^{1/3}\gg1. \tag{18}$$

This 1/2 exponent is three times bigger than the 1/6 in Eq. (12) so it should be much more easily measured in experiment. $\hat{\xi}$ is not forgotten by the final configuration because the negative domains are $\hat{\xi}$-sized islands in the positive sea.

Fully nonlinear numerical simulations in one spatial dimension are consistent with the exponent 1/2 as shown in Fig. 1. The initial $\hat{\xi}$-sized seeds and the two final negative $\hat{\xi}$-sized domains are shown in Fig. 2.

B. Transition through bubble nucleation

The uniform $\phi=M$ is stable against small perturbations as long as $\epsilon<3M^2$. However, already for $\epsilon>M^2$, $\phi=M$ is between the two minima $\pm\sqrt{\epsilon}$ of the double-well potential $V(\phi)$. For $M^2<\epsilon<3M^2$ the uniform $\phi=M$ is metastable:

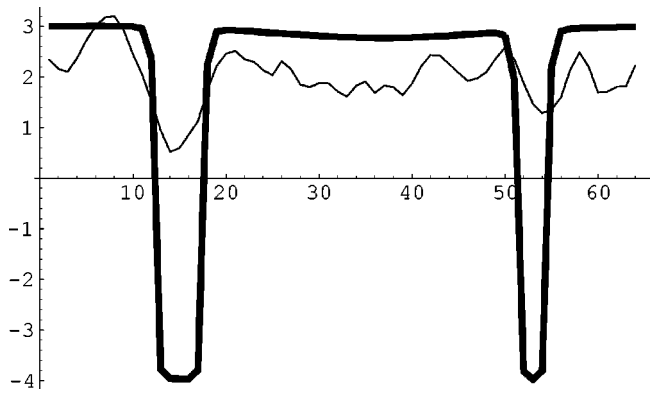

FIG. 2. Two snapshots of ϕ as a function of x for $M=2, \tau=128$, and $T=10^{-5}$ taken from numerical simulations. The thin line is ϕ at $t=0.8\hat{t}$ when spinodal decomposition begins (the amplitude of fluctuations around $M=2$ is magnified 100 times). The thick line is ϕ at $t=1.6\hat{t}$ when kinks are already well defined.

ϕ conservation does not forbid its decay into a nonuniform configuration with negative bubbles scattered in the positive phase. The decay can in principle proceed by nucleation of bubbles of the negative phase. The nucleation is most likely to occur in one spatial dimension. Therefore, we restrict our analysis to one dimension where the bubble is an antikink-kink pair (AKP).

First of all we will estimate the minimal temperature T necessary to nucleate an AKP. The effective potential $V(\phi)$ can be approximated for small fluctuations around $\phi=M$ by $\frac{1}{2}\varepsilon\tilde{\phi}^2$, where $\varepsilon=\epsilon-3M^2$. Magnitude of the fluctuations around M is given by $\langle\tilde{\phi}^2\rangle\sim T/\sqrt{|\varepsilon|}$. The fluctuations around $\phi=M$ can result in AKP nucleation if they can reach beyond the positive inflection point at $\phi=\sqrt{(3M^2+\varepsilon)/3}$ or

$$\left(M-\sqrt{M^2+\frac{\varepsilon}{3}}\right)^2\approx\langle\tilde{\phi}^2\rangle. \tag{19}$$

To first order in $|\varepsilon|/M^2$ nucleation can take place for $\varepsilon>-(TM^2)^{2/5}\equiv-\varepsilon_n$. According to this estimate, nucleation becomes possible when $\epsilon(t)$ in Eq. (14) is approaching the critical $3M^2$ from below.

In fast transitions there may be not enough time for the nucleation to actually happen. We will estimate the nucleation time as follows. The CHC equation at $\varepsilon=-\varepsilon_n$ can be approximated by

$$\partial_t\tilde{\phi}=-\nabla^4\tilde{\phi}+\epsilon_n\nabla^2\tilde{\phi}+\vec{\nabla}\vec{\xi}. \tag{20}$$

The relaxation time at $k_n=\sqrt{\varepsilon_n}$, which corresponds to the correlation length, is $\sim\varepsilon_n^{-2}$. ε_n^{-2} is the time scale for AKP nucleation when $\epsilon=3M^2-\varepsilon_n$. At a later time and ϵ closer to the critical $3M^2$ the nucleation time is longer. If the nucleation time ε_n^{-2} is longer than the time $\varepsilon_n\tau$ remaining to the transition [the time $\epsilon(t)$ needs to grow from $3M^2-\varepsilon_n$ to $3M^2$], then there is not enough time for any AKP's nucleation. This condition is satisfied when

$$\tau(TM^2)^{6/5}\ll 1. \tag{21}$$

If the transition is fast enough or T is sufficiently small, then no AKP are nucleated while $M^2<\epsilon(t)<3M^2$ and ϕ remains fluctuating around uniform $\phi=M$ until $\epsilon(t)$ crosses $3M^2$. If the condition (21) is satisfied, then the transition proceeds by spinodal decomposition and results with negative $\hat{\xi}$-sized bubbles. If the opposite condition holds, then the outcome is a finite density of negative bubbles with a size independent on τ.

It is known that many-kink effects can play a role in AKP nucleation [5]. In our case these effects are irrelevant because we study nucleation from a uniform $\phi=M$ phase, which is initially free of any kinks.

IV. INHOMOGENEOUS TRANSITION

A uniform phase transition (3) may be a good first approximation in some cases but in real life we often have to face the fact that it is not perfectly homogeneous. To gain some insight, let us begin with the temperature front of the form

$$\epsilon(t,x)=\begin{cases}+\epsilon_-, & x<vt\\ -\epsilon_+, & vt<x,\end{cases} \tag{22}$$

in the limit of very slow v. We will argue *a posteriori* that the sharp step is a good approximation of any generic front for $v\to 0$.

We solve the problem of kink generation behind the moving front by perturbative expansion around $v=0$. At $v=0$ there is a static ϕ front, $\phi(x)=H(x)$—a step in ϕ at $x\approx 0$ interpolating between $-\sqrt{\epsilon_-}$ at $x=-\infty$ and 0 at $x=+\infty$. Its width is proportional to $\epsilon_-^{-1/2}+\epsilon_+^{-1/2}$.

Let us now switch on a small $v>0$. The ϵ front Eq. (22) is slowly moving on. If ϕ were not conserved, the ϕ front would follow moving in step with ϵ front leaving no kinks behind [7]. For our conserved ϕ this is not possible, kinks must inevitably appear. To see in detail how it happens, let us substitute $\phi(t,x)=H(x-vt)+\psi(t,x)$ with $\psi=O(v)$ to Eq. (1) and keep only $O(v)$ terms. We are interested in large length scales as compared to the width of the step $H(x)$ and that of the ϵ front. That is why we keep only up to the second x derivative, which is responsible for diffusion. Far from $x\approx vt$ we obtain

$$\dot{\psi}(t,y)=\epsilon_+\psi''(t,y)+v\psi'(t,y)+v\sqrt{\frac{\epsilon_+}{2}}\theta(t)\delta(y), \tag{23}$$

where $y=x-vt$. We take into account a δ-like source term at $y=0$ that is a long-wavelength approximation to vH'. We also set $\epsilon_-=\epsilon_+/2$ for simplicity. The source term is switched on at $t=0$ when the ϵ front starts to move, hence the Heaviside function. The solution of Eq. (23) is straightforward,

$$\psi(t,y)=v\sqrt{\frac{\epsilon_+}{2}}\int_0^t dt'\frac{\exp(-[y+v(t-t')]^2/4\epsilon_+(t-t'))}{\sqrt{4\pi\epsilon_+(t-t')}}. \tag{24}$$

The source term produces ψ at $y=0$ at a constant rate. It spreads around by diffusion but at the same time it is carried to negative y with velocity $-v$. For $y>0$, the diffusion dominates at first but at $v^2t^2 \sim \epsilon_+ t$ the two processes balance one another and $\psi(t,y>0)$ saturates. From this time on all the ψ is carried directly from the source to $y<0$ with velocity $-v$. This means that the ϕ front is halted, while the ϵ front keeps moving on. A supercooled phase with a slightly positive ϕ is growing in between them with velocity v. When its width exceeds $\sqrt{2/\epsilon_+}$, ϕ decays towards positive ground state. From this time on, we have a negative ϕ step moving together with ϵ step and the whole story repeats itself at spatial intervals of ϵ_+/v. Density of kinks is

$$n \sim \frac{v}{\epsilon_+} \quad \text{for } v \to 0. \tag{25}$$

It should be stressed that the whole process is deterministic; kinks are made at regular intervals. Noise is required to begin the process; it also adds some irregularity on top of the regular pattern.

Note that for small v, the relevant length scale is ϵ_+/v. For small enough v, it far exceeds the ϵ-front width and the width of $H(x)$. This justifies the sharp step in Eq. (22) and the long-wavelength approximations involved in our derivation of Eq. (23).

Let us now turn to the opposite large-v limit where we anticipate the transition to be effectively homogeneous. Any generic $\epsilon(t,x)$ can be linearized around $\epsilon=0$,

$$\epsilon(t,x) = \frac{vt-x}{v\tau} \equiv \alpha(vt-x). \tag{26}$$

At any fixed x, the transition proceeds at the rate of $1/\tau$ just, as in Eq. (3). If it were homogeneous it would enhance the momentum $\hat{k}=\tau^{-1/6}$. For

$$v \gg \alpha^5 \quad \text{or} \quad v \gg \tau^{-5/6}, \tag{27}$$

this momentum scale is much bigger than the slope α and the relevant field fluctuations do not feel the inhomogeneity. This is where the transition is effectively homogeneous and Eq. (12) applies.

V. CONCLUDING REMARKS

Our calculations predict domain wall density at the end of the spinodal decomposition stage. This is the initial density for phase-ordering kinetics (POK) process described in Ref. [8].

For binary fluids in more than one dimension hydrodynamic processes are known to be important for phase-ordering kinetics, see, e.g., [9]. Hydrodynamics can be introduced by coupling the CHCE to the Navier-Stokes equation (NSE) plus the incompressibility condition, $\nabla\vec{v}=0$. Order parameter fluctuations induce hydrodynamic flows. It has to be kept in mind that before the spinodal decomposition blows up, the fluctuations are small. According to NSE, the induced velocity is formally proportional to the fluctuation squared. When this velocity is substituted back to CHCE, it gives a term that is formally cubic in order parameter fluctuations. This term has the same formal order of magnitude as the usual nonlinear term in our Eq. (1). For the same reasons it can be neglected at the onset of the spinodal decomposition. If the viscosity of the fluid is large, then the usual cubic term dominates. The domain wall structure at the end of spinodal decomposition is the same as if there were no hydrodynamic flows at all. If, on the other hand, viscosity is small, then the ''hydrodynamic'' cubic term is the dominant one. It halts the spinodal decomposition somewhat earlier than it would be halted by the standard nonlinearity. This can give only negligible logarithmic corrections to $\hat{k}$ because $\hat{t}$ [see Eq. (10)], is the time of the double exponential blow-up in the power spectrum of Eq. (9). The ordered domains at the end of the spinodal decomposition, at $\hat{t}$, are metastable [10]. $1/\hat{k}$ gives the initial size of the domains in this metastable state. In both large and small viscosity limits the correlations at the end of spinodal decomposition stage are determined by $\hat{k}$. Hydrodynamics is irrelevant when initial density of domain walls is considered.

Hydrodynamics is even less relevant in an effectively one-dimensional binary fluid system. This limit can be achieved for binary fluids trapped in a thin tube whose diameter is less than $1/\hat{k}$. In one dimension the incompressibility condition $\nabla\vec{v}=0$ makes impossible any nontrivial flow and the model reduces to just CHCE.

In one dimension, kinks can be thermally activated at arbitrarily low temperatures. Still, our scaling relations hold if the freeze-out at $\hat{t}$ takes place outside the Ginzburg regime, where the thermal kink nucleation is exponentially suppressed. If the freeze-out happens above the Ginzburg temperature, then there is possibility of crossover: $\hat{\xi}$ does not diverge with $\tau \to \infty$ but saturates at a finite value. This case is interesting in its own right but requires further investigation.

It should be possible to test our prediction in a two-dimensional experiment. In fact, most of the experimental work on POK in binary fluids was done in two dimensions (2D). In 2D it may be difficult to distinguish between spinodal decomposition stage and POK stage. However, the scaling exponents for early POK are known and can be used to extrapolate the length scale back to $\hat{t}$. An extrapolation like this was used in numerical simulations of vortex formation in 2D, see Fig. 3 in Ref. [11].

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